Theory of Intergenerational Transfers in a Macro Context

Ronald Lee, October 17, 2005
Tokyo
1 Introduction: The age dimension

There is an important age dimension to economic activity, and it is mostly ignored by economists and economic accounting systems.

Flows of income from individuals of one age to those of another age are a pervasive part of our everyday life, but from the point of view of government statistics they are largely invisible.
Much of economic output is not consumed by the person whose labor produced it. Of course, it may be saved and consumed later. More typically it is given, or transferred, to someone else. For example, a parent earns income and uses it to raise her children. Or to care for an elderly parent. Or to pay taxes that support government transfer programs for pensions, healthcare, public education, or to alleviate poverty.
Our project is mapping the age dimension of income flows through the economy in a comprehensive way.

When these flows are considered individually, often a misleading picture emerges, because there are substitutions and crowding out.

It is important to look at the complete picture.

This can be complicated, because the flows are mediated by different institutions, particularly the family, the market and the state, and take different forms.
This morning, I will try to sketch some of the underlying theory, and show how these flows can be studied systematically and related to the macro economy.

This afternoon, I will talk more specifically about transfers through the public sector in the US, and show several concrete applications of the National Transfer Accounts to important policy questions.
References for this talk


2. The economic life cycle: Labor earnings and consumption

- People produce output over their life cycles, say $y_l(x)$ at age $x$.
- People need or desire to consume output over their life cycles, driven by biology, culture, technology, optimization, say on average $c(x)$.
- $y_l(x)$ and $c(x)$ do not exactly correspond over the life cycle
  - Children produce little or nothing, but consume
  - Elderly may produce little but consume.
- Various mechanisms and institutions fill in the gaps by reallocating across age.
Stylized human life cycle for individuals (conditional on survival)

Output per person per year

Labor earnings, \( y_l(x) \)
Consumption, \( c(x) \)

Age

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How do these reallocations across age and time take place?

- By institution, through the family, the market and the state (public sector)
- By mechanism, through borrowing and lending, through investing in physical capital, and through transfers.
- So three institutions, three mechanisms: construct a 3X3 table as in next slide.
# Resource Reallocation Across Age and Time

<table>
<thead>
<tr>
<th>Form of Reallocation</th>
<th>Institution</th>
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<td><strong>Borrowing/ Lending</strong></td>
<td>Familial Loans</td>
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<td>&quot;Transfers&quot;</td>
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How can we measure and interpret this rich variety of resource flows across age and time?

• One strategy: measure each of these flows in detail, which requires:
  – Rich data from many sources
  – Strong assumptions, in some cases
  – But is surprisingly possible.

• Another strategy: examine broad patterns of production and consumption, and infer the flows that must have occurred.

• I will show methods and results for both approaches, time permitting, but start with second.
Illustration: US Consumption and Labor Earnings by Age in 2000 (includes public educ, health care, other in-kind transfers)
What are these age profiles? Andy will give more detail later, but…

- Earnings are averaged over all individuals, whether working or not
  - Pretax
  - Include employer provided benefits
  - Include self employment (entrepreneurial income)

- Consumption
  - All private expenditures (imputed by age within household)
  - All public in-kind transfers: education, health care, etc.
  - Per capita share of all other govt expenditures such as defense, NIH, roads, police
US Consumption and Labor Earnings by Age in 2000 (includes public educ, health care, other in kind transfers)

Net cost per child is less than net cost per elder if we count only labor earnings.

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US Consumption and Labor Earnings by Age in 2000
(includes public educ, health care, other in kind transfers)

Earnings exceed consumption only from age 26 to 57

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Why do labor earnings decline so early in life, in the 50s?

- US life expectancy is 30 years higher now than in 1900, and older people are in much better health, yet
- Retirement age has declined strongly
  - In 1900, men retired at 74 (median)
  - Today, at 63
- Similar story in European countries
- What drove this decline?
  - Rising wealth and preference for leisure
  - Pension programs helped workers implement their preferences
  - Public and private pension programs distorted incentives for early retirement, particularly in Europe.
More detail on consumption, for US and Taiwan

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3. Theory for the Golden Rule
Steady State Case: Flows

- Steady state ("stable") populations
  - Population age distribution is constant over time
    - Population growth rate = $n$
  - Age distribution is proportional to $e^{-nx}l(x)$ where
    $l(x)$ is proportion surviving from birth to age $x$.
    - Rapid growth (high $n$) means young population
    - For given $n$, higher $l(x)$ (lower mortality) means older
      population.
    - Note formal similarity to discounting expected future
  - Higher fertility always makes pop younger
  - Lower mortality raises growth rate, raises survival,
    effect can be younger or older, depending on initial level.
Golden Rule Theory (cont.)

• Golden rule steady state means that
  – Discount rate = rate of return to capital = population growth rate plus productivity growth rate
    • For simplicity, take $r = n$; prod gr = 0.
  – All of labor earnings are consumed; all of capital income is saved.

• Consider age profiles of consumption and labor earnings in a given year for this economy.
  – By assumption, aggregate consumption equals aggregate labor earnings.
This aggregate constraint can be written

$$\int_{0}^{\omega} e^{-nx} l(x) \left[ y_l(x) - c(x) \right] dx = 0$$

$$Y_l = C$$

• Total population-weighted production must equal total population-weighted consumption (golden rule: cons = labor earnings).
In golden rule steady state, cross-section and life cycle are similar

\[ C = \int_0^\omega e^{-rx} l(x) c(x) dx \]

- Present value of life cycle consumption at birth

\[ C = \int_0^\omega e^{-nx} l(x) c(x) dx \]

- Aggregate consumption (sum of c(x) times share of population at age x)

- When \( r=n \) (golden rule), these are identical

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So $C = Y_t$ implies that the life cycle budget constraint also holds

• $C = Y_t$ implies that the present value of life time consumption equals the present value of life time labor earnings, in golden rule ($r=n$).
Average ages of consumption and labor earning

\[ A_c = \frac{\int_0^\omega x e^{-nx} l(x)c(x) \, dx}{\int_0^\omega e^{-nx} l(x)c(x) \, dx} \]

- Numerator is the amount consumed in the population age \( x \) times age \( x \)
- Denominator is total amount consumed
- Ratio is the age at which the average dollar is consumed in the population.

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Is income redistributed upwards or downwards across age, on average?

- Caldwell has argued that “wealth flows” are upwards in traditional societies, from children to adults.
- Others have argued that wealth (income) flows downwards on net, from adults to children.
- Assess by calculating the average age at which a dollar is consumed in the population, and similarly at which one is produced, \( A_c \) and \( A_y \).
Direction of wealth flows (cont.)

• Now compare the two.
  – If $A_c < A_y$, then consumption precedes production on average, so income is redistributed downwards across ages, from old to young.
  – If $A_y < A_c$ then the reverse: the young are transferring to older individuals on average.
Stylized human life cycle for surviving individuals

Here, average consumption is later than average production, so wealth flows upwards.

Average dollar is consumed by someone older than the person who earned it.
Some estimated average ages for very different economic/cultural regimes

- Estimated from very different kinds of data
- Seem to tell a very consistent story
Figure 2. Comparative Direction of Reallocation of Income

- Household Data
  - Ac - Ay (in years)
    - +5
    - +7
    - +6
    - +3
    - -2

- Individual Data
  - -6
  - -4
  - -4
  - -7
  - -11

- Industrial Welfare State
  - England [Ermisch]
  - Japan [Ermisch]
  - United States [Lee & Miller]
  - 2050

- Intensive Agriculture
  - India & Egypt [Mueller]

- Extensive Agriculture
  - Cote D'Ivoire [S tecklov]
  - Maya [Lee & Kramer]

- Hunter-gatherer
  - Amazon [Dodds, et. al., pooled data]
  - Amazon [Kaplan, pooled data]
4. Theory for the Golden Rule
Steady State Case: Stocks of wealth or debt
Suppose babies had to go into debt to feed themselves....

- At the start of life, \( c(x) > y_l(x) \); dependency.
- Suppose we keep a notional account of debt and credit over the life cycle, discounted to age 0.
- Credit gained (or lost) at age \( x \) is:

  \[
  e^{-rx} l(x) \left[ y_l(x) - c(x) \right]
  \]

  where \( r \) is interest rate, \( l(x) \) is survival from 0 to age \( x \). Credit rises when \( y_l(x) > c(x) \)
- Cumulated up to age \( x \), we get \( W(x) \):

  \[
  W(x) = \int_0^x e^{-rx} l(x) \left[ y_l(x) - c(x) \right] dx
  \]
Here is how the flows correspond to the stock of wealth by age

• For now I will simplify, by considering the special case in which \( r = n \), in which the interest rate equals the population growth rate. “Golden rule” case for economic growth.

• More general analysis has been done, but more complicated (see Bommier and Lee, 2003).
Stylized human life cycle for surviving individuals

Labor earnings, $y_l(x)$
Consumption, $c(x)$

Wealth (+) or Debt (-), $W(x)$

Output per person per year

Output per person

AGE

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$W(x)$, a stock, is the present value at birth of $y_l(a)-c(a)$ up to age $x$, and weighted by the probabilities of survival to age $a$.

Note that $W(x)$ reaches min and max when $y_l(x)$ crosses $c(x)$ and they are equal.
Interpretation of life cycle wealth at age x, W(x)

- W(x) is amount of wealth (or debt) a person would necessarily have at age x if that person had the age profiles of consumption and production c(x) and y_l(x).
- Alternatively, W(x) is the amount of wealth at age x that a person would demand (need) if she planned to have life time labor income y_l(a) and desired a life time consumption trajectory c(a), for trajectories satisfying the life cycle budget constraint with r=n. (e.g. with optimal consumption plan).
The average wealth per capita in the population may be pos or neg

- The red line in last slide plotted \( W(x) \), the amount of life cycle wealth or debt per individual age \( x \).
- Now find the average level of wealth per capita in the whole population, call it \( W \).
- \[ W = \frac{\text{pop}(x) \times W(x)}{\text{totpop}} \]

\[
W = \frac{\int_{0}^{\omega} e^{-rx} l(x)W(x) \, dx}{\int_{0}^{\omega} e^{-rx} l(x) \, dx}
\]

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Stylized human life cycle for surviving individuals

Output per person per year

\[ y(x) \]
\[ c(x) \]
\[ W(x) \]

Population weighted average per capita wealth \( W \) in pop; could be pos or neg.

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An elegant result

- If $A_c > A_y$ then indivs want to hold onto some output for later consumption, or accumulate claims on later production. This means wealth, $W$, is on average positive in the population.
- If $A_c < A_y$ then people want to consume before they produce, and must go into debt on average, so $W$ is negative.
- The surprising mathematical result:
  \[ W = c(A_c - A_y) \], where $c$ = per capita cons
Stylized human life cycle for surviving individuals

Here, average consumption is later than average production, so wealth flows upwards.

Population weighted average per capita wealth $W$ in pop; could be pos or neg. Here, pos.

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Stylized human life cycle for surviving individuals

Output per person per year

Here, average consumption is later than average production, so wealth flows upwards.

The area of the arrow = W, direction of arrow shows upward. This equals height of green line.

Output per person

Population weighted average per capita wealth W in pop; could be pos or neg. Here, pos.

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What determines $W$?

- Depends strongly on the population age distribution.
  - A young population, with a high proportion of dependent children ($y_1(x) < c(x)$), will have smaller $W$, or negative $W$.
  - An old population, with a high proportion of dependent elderly, will have a larger $W$, since the elderly must have positive wealth.
  - Also depends on the shapes of the age profiles of earning and consuming
    - Are there large redistributions to children? Investments in human capital?
    - Are there large redistributions to the elderly for consumption or for health care?
5. Economic consequences of population aging (mort const)

• Choice of consumption path $c(x)$ is subject to a budget constraint: Aggregate consumption cannot exceed aggregate production.

• Suppose $y_l(x)$ remains the same, but the population growth rate increases.
  – Then the stable pop age distr gets **younger**. Dependency ratios change.
  – This changes the amount of aggregate consumption and production associated with $c(x)$ and $y_l(x)$.
  – Previous balance no longer holds.
  – Adjustment in either consumption or earnings is necessary to keep within the budget constraint. Assume consumption adjusts
The “social budget constraint” or aggregate budget constraint is:

\[ \int_0^\omega e^{-nx} l(x)\left[ y_l(x) - c(x) \right] dx = 0 \]

• Total population-weighted production must equal or exceed total population-weighted consumption (golden rule: cons = labor earnings).

• Now we will differentiate it with respect to n, in the next slide, and set this derivative also = 0.
Differentiating the aggregate budget constraint

\[ \int_0^\omega e^{-nx} l(x) \left[ y(x) - c(x) \right] dx = 0 \]

• Aggregate budget constraint

\[ \int_0^\omega xe^{-nx} l(x) \left[ y(x) - c(x) \right] dx \]

• Differentiate it with respect to \( n \)

\[ + \int_0^\omega e^{-nx} l(x) \left[ \frac{dy(x)}{dn} - \frac{dc(x)}{dn} \right] dx = 0 \]

• Assume \( y_l(x) \) does not change, so \( dyl(x)/dn = 0 \), and all adjustment is in \( c(x) \)
Differentiating the aggregate budget constraint (continued)

- Divide through by $C$ (=Y), aggregate consumption and prod.
- Recall definitions of average ages.

\[
\left( \frac{dC / C}{dn} \right) = A_c - A_y
\]
How can we interpret this result?

\[
\left( \frac{dC}{C} \right) = \frac{dn}{dn} (A_c - A_y)
\]

- When \( n \) changes by \( dn \), the present value of life time consumption changes by a proportion equal to \( dn(A_c - A_y) \)
  - Suppose \( dn = +.01 \) (that is, pop growth rises by 1% per year)
  - Suppose \( A_c = 49 \) and \( A_y = 44 \), so \( A_c - A_y = 5 \)
  - Then life time consumption rises by 5%
    - E.g. \( c(x) \) can increase by 5% at every age
    - Some other pattern of change is also possible.
- If \( A_c - A_y > 0 \) (wealth flows upwards), then more rapid growth is beneficial. If it flows downwards, then less rapid growth is beneficial.
  - Industrial nations are upwards, Third World is downwards.
Labor can adjust too

• Now suppose that all the adjustment is in labor supply or productivity, \( y_l(x) \), while \( dc(x)/dn = 0 \).

• Now with previous setup, the present value of life time production can decrease with more rapid pop gr, because there are relatively fewer elderly in need of upward transfers, and more young to share burden.

• For example, if people retired after 40 years of work initially, after \( n \) increased they could work 5% less by retiring 2 years earlier (after 38 years) or by working 2 hours less per week, etc.

• For a reduction in population growth, as actually is occurring in developed countries, all this is in reverse.
6. Adding detail to the basic model
So far, we have not considered how income is reallocated

• So far, just comparing $c(x)$ and $y_1(x)$
• Now consider mechanisms in more detail
• Rewrite the individual budget constraint for age $x$
Extending the analytic framework

• The budget constraint of an individual age x can be spelled out in more detail.

• Notation:

  \[ \tau(x) = \text{net transfers received (neg if transfers given)} \]
  \[ m(x) = \text{net borrowing (neg if loans made) (inflow for M)} \]
  \[ i(x) = \text{net investing (neg if investments sold) (inflow for K)} \]
  \[ K(x) = \text{physical capital stock owned per original member of birth cohort} \]
  \[ M(x) = \text{net credit (money owed to) including financial capital, bonds, etc.; negative if debt is owed on net.} \]

\[ 0 = y_t(x) + \tau(x) + m(x) - c(x) - i(x) + n\left[ K(x) + M(x) \right] / l(x) \]
Stocks: W, K, M, T

- As noted earlier, W can be pos or neg. It is average wealth needed for population to achieve its desired c(x) trajectory given y_l(x).
- K is average physical capital per capita, as in standard macroeconomic models.
- M is average amount of credit or debt held in the population. In financially closed population, M=0, because every dollar of credit is a dollar of debt for someone else; it all sums to 0. (Assumed closed economy)
- T is average transfer wealth per capita. It can be positive or negative. It is a social construct, created by rules and promises, with no physical counterpart. Example: “Social Security Wealth”, which equals “Implicit Debt”, or about 11 trillion dollars currently in US.
A fundamental result

\[ W = K + T \]

Life cycle wealth can be held in the form of physical capital or transfer wealth, and these substitute for one another in satisfying the demand for wealth.

This is background for worry that PAYGO pension programs (transfer programs) undermine capital accumulation. They create transfer wealth which substitutes for real wealth, that is capital.

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Making the accounts more detailed

• Above, I introduced transfers, investments, and loans.
• Each of these could be distinguished by institution: family, market or government.
• We could further distinguish kinds of transfers within each institution: pensions, health care, education, etc.
Relaxing the assumptions

• Those nice mathematical results hold for the golden rule case.
• For general steady state case, many nice results too
• For actual NTA, we do not assume golden rule or any kind of steady state.
  – Use actual variation over age and time in interest rates, productivity, demography, programs, etc.
  – Golden rule case just helps to develop insight.
END

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